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B.Sc-part-II(H). paper-III. Date-03-05-2020.
Differential Calculus (curvature)

Q.1 For the cardioide $r = a(1-\cos\theta)$, prove that the chord of curvature through the pole is $\frac{4}{3}r$.

Sol^n We have, $r = a(1-\cos\theta)$ ——— (I)

Differentiating w.r.t. to θ, we get

$\frac{dr}{d\theta} = a\sin\theta$. ——— (II)

We know that

$$\tan\phi = r\frac{d\theta}{dr} = \frac{a(1-\cos\theta)}{a\sin\theta} \quad [\text{from I and II}]$$

$$\Rightarrow \tan\phi = \frac{\cancel{2}\sin^2\frac{\theta}{2}}{\cancel{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \tan\frac{\theta}{2}$$

$$\Rightarrow \tan\phi = \tan\frac{\theta}{2} \Rightarrow \phi = \frac{\theta}{2} \quad \text{——— (III)}$$

Also, we have $p = r\sin\phi = r\sin\frac{\theta}{2}$ [from III]

$$\Rightarrow p = r\sin\frac{\theta}{2} = r\sqrt{\sin^2\frac{\theta}{2}} = r\sqrt{\frac{1-\cos\theta}{2}}$$

$$= r\sqrt{\frac{r}{2a}} \quad [\text{from (I)}]$$

$$p = \frac{r^{\frac{3}{2}}}{\sqrt{2a}} \quad \text{——— IV}$$

Differentiating w.r.t. r, we get

$$\frac{dp}{dr} = \frac{3}{2}\frac{r^{\frac{3}{2}-1}}{\sqrt{2a}} = \frac{3}{2}\frac{r^{\frac{1}{2}}}{\sqrt{2a}} \quad \text{——— V}$$

Hence, the chord of curvature through the pole $= 2p\frac{dr}{dp}$

$$= 2\times\frac{r^{\frac{3}{2}}}{\sqrt{2a}}\times\frac{2}{3}\times\frac{\sqrt{2a}}{r^{1/2}} \quad [\text{from IV and V}]$$

$$= \frac{4}{3}r^{\frac{3}{2}-\frac{1}{2}} = \frac{4}{3}r$$

Proved.

B.Sc-part-II(H), paper-III, date-03-05-2020

Q.2. Find the chord of curvature through the pole of the cardioide $r = a(1+\cos\theta)$

Soln: We have $r = a(1+\cos\theta)$ ——— (I)

Differentiating w.r.t. θ, we get

$$\frac{dr}{d\theta} = -a\sin\theta \text{ ——— (II)}$$

We know that $\tan\phi = r\frac{d\theta}{dr}$

$$\Rightarrow \tan\phi = \frac{a(1+\cos\theta)}{a\sin\theta} \quad [\text{from I and II}]$$

$$\Rightarrow \tan\phi = -\frac{\cancel{2}\cos^2\frac{\theta}{2}}{\cancel{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = -\cot\frac{\theta}{2} = \tan\left(\frac{\pi}{2}+\frac{\theta}{2}\right)$$

$$\Rightarrow \cancel{\tan}\phi = \cancel{\tan}\left(\frac{\pi}{2}+\frac{\theta}{2}\right)$$

$$\Rightarrow \phi = \frac{\pi}{2}+\frac{\theta}{2}$$

Also, we have, $p = r\sin\phi = r\sin\left[\frac{\pi}{2}+\frac{\theta}{2}\right]$

$$\Rightarrow p = r\cos\frac{\theta}{2} = r\sqrt{\cos^2\frac{\theta}{2}} = r\sqrt{\frac{1+\cos\theta}{2}}$$

$$= r\sqrt{\frac{r}{2a}} \quad [\text{from I}$$

$$\Rightarrow p = \frac{r^{3/2}}{\sqrt{2a}} \text{ ——— III}$$

Differentiating w.r.t. r, we get

$$\frac{dp}{dr} = \frac{3}{2}\frac{r^{\frac{3}{2}-1}}{\sqrt{2a}} = \frac{3}{2}\frac{r^{\frac{1}{2}}}{\sqrt{2a}} \text{ ——— (IV)}$$

Hence, chord of curvature through the pole

$$= 2p\frac{dr}{dp} = 2\times\frac{r^{\frac{3}{2}}}{\cancel{\sqrt{2a}}}\times\frac{2}{3}\times\frac{\cancel{\sqrt{2a}}}{r^{\frac{1}{2}}} \quad [\text{from III and IV}]$$

$$= \frac{4}{3}r^{\frac{3}{2}-\frac{1}{2}} = \frac{4}{3}r$$

Solved.

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